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The period and amplitude changes of Polaris (α UMi) from 2003 to 2007 measured with SMEI

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ABSTRACT

We present an analysis of 4.5 years of high precision (0.1%) space-based photometric measurements of the Cepheid variable Polaris, obtained by the broad band Solar Mass Ejection Imager (SMEI) instrument on board the Coriolis satellite. The data span from April 2003 to October 2007, with a cadence of 101 minutes and a fill factor of 70%. We have measured the mean peak to peak amplitude across the whole set of observations to be 25 mmag. There is, however, a clear trend that the size of the oscillations has been increasing during the observations, with peak to peak variations less than 22 mmag in early 2003, increasing to around 28 mmag by October 2007, suggesting that the peak to peak amplitude is increasing at a rate of 1.39 ± 0.12 mmag yr^{-1}. Additionally, we have combined our new measurements with archival measurements to measure a rate of period change of 4.90 ± 0.26 s yr^{-1} over the last 50 years. However, there is some suggestion that the period of Polaris has undergone a recent decline, and combined with the increased amplitude, this could imply evolution away from an overtone pulsation mode into the fundamental or a double pulsation mode depending on the precise mass of Polaris.

Key words: stars: Cepheids – stars: pulsation

1 INTRODUCTION

In spite of Julius Caesar's view (Shakespeare 1623, Act 3 Scene I), Polaris is in fact one of the more inconstant of stars. In addition to not being precisely located at the North Celestial Pole, it is a variable star, with a pulsation period of nearly 4 days and a current pulsation amplitude of around 30-50 mmag in the V band. Additionally, Polaris is not even constant in its inconstancy, as both the pulsation period and the pulsation amplitude have changed in the past. The amplitude in particular has changed substantially and is currently still changing, as we will describe in this paper.

Polaris is an important star for a number of reasons – it is the nearest Cepheid variable and a star where we can see stellar evolution taking place. Understanding the location of Polaris on the Hertzsprung-Russell diagram (and particularly the relationship of Polaris to the Instability Strip; whether it is a star undergoing its first or third or even fifth crossing) and the nature of the pulsations (whether fundamental model or an overtone pulsator) are all important questions in stellar evolution.

The first evidence suggesting the variable nature of Polaris was presented 150 years ago (Seidel 1852; Schmidt 1857) with strong confirmation, along with the correct period, being supplied by Campbell (1899) via radial velocity measurements. Photometric detection of the pulsations were presented several years afterwards (Hertzsprung 1911; Pannekoek 1913). A large number of observations have have been made in the intervening years, which have helped to build a picture of how the star has been evolving over the last century and a half. Despite this, there is still a great deal of interest in Polaris due to the changes in the period and amplitude of the oscillations, as well as unusual events such as the change from a steady decline in amplitude to a very rapid decline from 0.1 mag to ∼0.02 mag during the 1960's. During this time, the period also readjusted downwards.

The rate of period change is an important diagnostic tool for determining which crossing of the instability strip a Cepheid is undergoing. The recent analysis of the O-C residuals of Polaris by Turner et al. (2005) has led to the suggestion that the period of Polaris is currently increasing at a rate of 4.5 s yr^{-1}. This rate of period change is unusual for a Cepheid with this period and adds to the confusion as to what stage of its evolution Polaris is actually at. In the following sections we present new SMEI photometric observations of Polaris, discuss the amplitude changes that we observe during the course of the observations and also look at the O-C residuals and interpret these in the light of recent measurements of the period.

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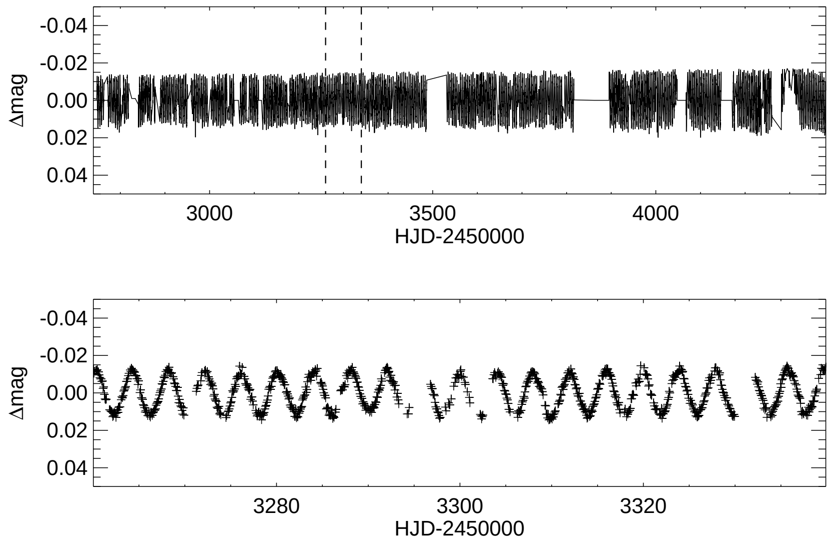

Figure 1. The complete 4.5 year SMEI time-series of Polaris is shown in the upper panel, whilst a short section from 13th September to 29th November can be seen in the lower panel. The relevant section of the full time series that has been depicted in the lower panel, is highlighted by the vertical dashed lines.

2 DATA PREPARATION AND ANALYSIS

The new photometry we present was obtained using the SMEI instrument on board the USAF Coriolis spacecraft, which was launched in January 2003 into an 840 km Sun-synchronous polar orbit with an orbital period of 101 minutes. SMEI consists of 3 cameras, each with a field of view of $60^\circ \times 3^\circ$, which monitors nearly the entire sky over one orbit. Consequently, we obtain data for Polaris on essentially every orbital pass. SMEI has a roughly triangular pass band with a peak quantum efficiency of 47% at 700 nm and falling to 5% at 430 nm and 1025 nm. Although SMEI is a small instrument, the fact that it has monitored the entire sky with a cadence of $\sim$ 100 minutes for over 4 years, results in stellar light curves, for bright stars, that are unprecedented. An overview of the SMEI instrument can be found in Eyles et al. (2003), and an overview of stellar variability results being obtained with SMEI will be presented in Spreckley & Stevens (2008). SMEI results on the variability of the Red Giant Arcturus can be found in Tarrant et al. (2007).

The Polaris data spans from April 2003 to October 2007, with a 70% fill throughout this period of time, giving us an exceptional data sample to investigate the period and amplitude variations of the 3.97 day oscillations exhibited by Polaris. The full details of the reduction pipeline for generating time-series from the SMEI data will be presented in Spreckley & Stevens (2008), so we only discuss the data reduction briefly here.

The raw images obtained by the SMEI instrument are bias subtracted, have a temperature scaled dark current signal removed, and are flat fielded. Hot pixels and high energy particle hits are corrected on the images via interpolation, before aperture photometry is performed. The resulting light curves are corrected for systematic effects resulting from the variation of the PSF as it moves across the CCD and vignetting/optical effects. Removing a best fit sine curve from the entirety of the dataset highlighted non-regular systematic variations at the few mmag level which we have largely removed using a smoothed box car average obtained with a window width of $\sim$12 days. We finally removed a number of spurious data points from the data, which were primarily due to uncorrected cosmic rays, by performing a 3 sigma clip on short 28 day sections of data from which the best fitting sinusoidal relation for each section had been removed.

The resulting time series can be seen in Fig. 1 along with a closer view of a section of data taken from 13th September 2006 to 29th November 2006, which highlights the level of precision we are able to attain over a long baseline.

3 RESULTS

In order to study the amplitude of the 3.97 day (2.914μHz) oscillation we have generated Fourier spectra of both long (8-9 months) and short (28 day) sections of the time series data. The resulting trend from computing the mean amplitude in each data chunk is shown in Fig. 2 and Fig. 3. To create the Fourier spectra, we first subtracted a robust mean from the time series and the residuals were then converted to a change in magnitude relative to the mean magnitude. After computing the Fourier spectra we ensured that in every case the power corresponding to the 3.97 day oscillation was restricted to a single bin, and padding the data with zeros to artificially enhance the resolution of the Fourier spectra had no effect on the computed amplitudes.

Fig. 2 shows the 2.914 μHz peak in the Fourier spectra for three $\sim$9 month sections of the light curve. The peaks for the 2003 and 2004-2005 data have been offset by -0.8 and -0.4 μHz respectively to highlight the increase in the amplitude over time. On the assumption that Polaris is oscillating in the first overtone mode (see section 4), we do not detect any significant power at the expected frequency for the fundamental mode, i.e. $\sim 2.1\mu$Hz. The trend of the increasing amplitude is highlighted more clearly in Fig. 3 where we have plotted the mean amplitude calculated in consecutive 28 day chunks of data. Each 28 day section contained 400 data points after data gaps had been filled with zero values.

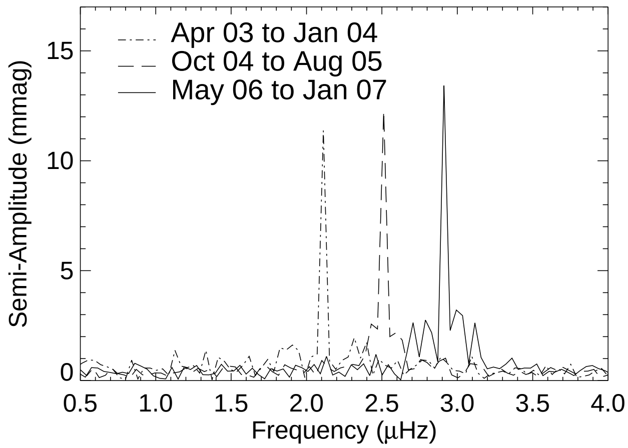

Figure 2. The Fourier spectra for 3 separate sections of the data, each spanning approximately 9 months. The spectra for April 2003 to January 2004, and October 2004 to August 2005 have been offset by 0.8 and 0.4 μHz respectively from the 2.914 μHz peak from the May 2006 to January 2007 data, to show the increasing amplitude. We find no evidence of a weak fundamental mode at $\sim 2.1\mu$Hz above the noise level in these spectra, with an amplitude above 1mmag.

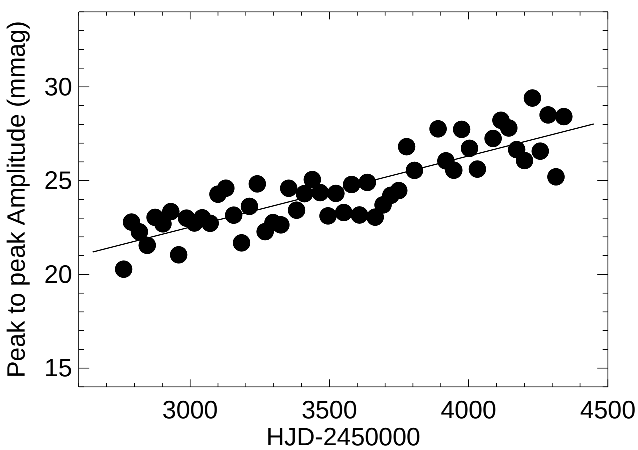

Figure 3. The mean peak to peak amplitude of the 2.914μHz (3.97 day) oscillations has risen at a rate of 1.39$\pm$0.12 mmag yr^{-1} over the last 4.5 years. The scatter exhibited about the mean trend is in part due to the fact that the amplitude of the oscillations of Polaris vary from cycle to cycle.

The best fitting linear relation describing the rate of increase for the peak to peak amplitude is 1.39 ± 0.12 mmag yr^{-1}. Some of the scatter in the plot is attributed to the varying amplitude of the oscillations, which Evans et al. (2004) also saw in WIRE observations, and suggested it could be due to the analogue of the Blazkho effect in Cepheids. In our analysis, however, we do not see any significant periodic variations in the amplitude above a level of ~ 1 mmag appearing consistently throughout the observations.

This is to our knowledge the first highly confident detection of the amplitude increase over the last few years from photometric measurements, although some hint has been given previously (Davis et al. 2003; Engle et al. 2004). This completely contradicts the claim that Polaris is about to cease its variability and leave the instability strip

Table 1. Recent measurements of the period of Polaris suggest that it has undergone another recent decline. There seems to have been a large decline between 1988 and 1993, with the rate slowing in the last ten years. In total, the period has reduced by around 200 seconds over the last 20 years.

Year	Period (days)	Reference
1987-1988	3.9746 ± 0.0008	Dinshaw et al. (1989)
1993-1994	3.97268 ± 0.00011	HC00
1994-1997	3.972352 ± 0.000003	HC00
		Kamper & Fernie (1998)
2003-2007	3.97209 ± 0.00004	This paper

(Dinshaw et al. 1989), although this was based somewhat on an erroneous result.

O-C residuals have been computed for each of the 28 day sections also. New times and phases of light maximum were determined by using the mean amplitudes calculated in the previous step to perform least squares fitting of the data, which has been re-phased to the period and epoch presented in Berdnikov & Pastukhova (1995):

$$\mathrm{HJD_{max}} = 2,428,260.727 + 3.969251E \tag{1}$$

where E is the number of elapsed cycles since this epoch. Examples of the phase folded data used to determine the O-C residuals are shown in Fig. 4. In this figure, the increase in amplitude over time, and the changing phase offset can be clearly discerned. The full set of O-C residuals are listed in Table 2.

The O-C residuals obtained from Turner et al. (2005), along with the new values calculated from our data are plotted in Fig. 5. We have used the same time regimes as this paper (i.e. pre-1963 and post 1965) to determine our rate of period change. The best fitting parabolic relation for observations before 1963, as determined by Turner et al. (2005), provides an estimate for the rate of period increase over this time of 4.44 ± 0.03 s yr^{-1} The best fitting parabolic relation for the data since 1963, with the inclusion of our new measurements suggests the mean rate of increase for the period has increased to 4.90 ± 0.26 s yr^{-1} This relation is again shown in Fig. 5. Ignoring the data from 1966, as in Turner et al. (2005), insignificantly alters the value to 4.99 ± 0.29 s yr^{-1}. Ignoring the datum from 1965, however, causes a dramatic change to the calculated value, giving instead 4.46$\pm$0.32 s yr^{1}. It is clear therefore that the mean rate of period increase over the last 50 years has been between 4.4 and 5 s yr^{-1}, consistent with the rate before 1963.

If one looks more closely at recent measurements for the period, however, it does appear that it may have recently undergone a rapid decline, similar to that seen in the early 1960's. Table 1 shows the period as measured several times over the last 20 years with the values obtained from Hatzes & Cochran (2000, hereafter HC00) and references therein, together with the period measured from our new results. The decline is very evident, and amounts to a decrease of around 200 seconds during the last 20 years, but the rate has been much slower over the last ten years than it was between 1987 and 1997. Additionally, although our results are fairly consistent with a period increase of

Table 2. Measurements of the times of maxima for the oscillations of Polaris, determined from fitting 1 month sections of data. N is the number of points used to compute each O-C value.

HJD	Cycle	O-C	N
2452763.768	6171	8.793 ± 0.055	198
2452787.637	6177	8.846 ± 0.040	311
2452815.417	6184	8.842 ± 0.071	170
2452855.113	6194	8.845 ± 0.063	235
2452870.954	6198	8.810 ± 0.085	182
2452902.800	6206	8.901 ± 0.036	333
2452930.578	6213	8.894 ± 0.040	310
2452962.393	6221	8.956 ± 0.051	220
2452986.213	6227	8.960 ± 0.046	271
2453017.990	6235	8.983 ± 0.039	302
2453041.822	6241	8.999 ± 0.032	252
2453077.577	6250	9.031 ± 0.035	205
2453097.476	6255	9.084 ± 0.030	283
2453133.211	6264	9.096 ± 0.037	295
2453157.013	6270	9.083 ± 0.048	344
2453184.832	6277	9.116 ± 0.053	305
2453212.657	6284	9.157 ± 0.046	287
2453240.456	6291	9.171 ± 0.064	307
2453268.224	6298	9.154 ± 0.031	333
2453296.050	6305	9.196 ± 0.044	275
2453323.862	6312	9.223 ± 0.036	326
2453355.648	6320	9.254 ± 0.047	299
2453383.429	6327	9.251 ± 0.038	335
2453411.268	6334	9.305 ± 0.035	280
2453439.071	6341	9.324 ± 0.034	353
2453466.878	6348	9.346 ± 0.034	319
2453482.754	6352	9.345 ± 0.087	73
2453534.432	6365	9.422 ± 0.067	61
2453550.291	6369	9.404 ± 0.032	276
2453582.042	6377	9.401 ± 0.032	354
2453609.845	6384	9.420 ± 0.039	282
2453633.720	6390	9.480 ± 0.036	290
2453661.498	6397	9.473 ± 0.029	306
2453693.278	6405	9.498 ± 0.029	352
2453721.077	6412	9.512 ± 0.029	353
2453748.891	6419	9.542 ± 0.027	285
2453776.690	6426	9.556 ± 0.027	307
2453808.445	6434	9.557 ± 0.033	170
2453899.794	6457	9.614 ± 0.127	114
2453915.709	6461	9.651 ± 0.044	347
2453947.502	6469	9.690 ± 0.034	291
2453975.325	6476	9.729 ± 0.043	343
2453999.122	6482	9.710 ± 0.035	225
2454030.908	6490	9.742 ± 0.028	373
2454086.529	6504	9.793 ± 0.030	349
2454118.265	6512	9.775 ± 0.028	323
2454138.145	6517	9.809 ± 0.033	208
2454177.925	6527	9.896 ± 0.040	147
2454197.761	6532	9.886 ± 0.031	343
2454225.598	6539	9.939 ± 0.104	268
2454253.344	6546	9.900 ± 0.083	207
2454289.133	6555	9.966 ± 0.071	125
2454316.910	6562	9.958 ± 0.053	223
2454340.754	6568	9.987 ± 0.026	342

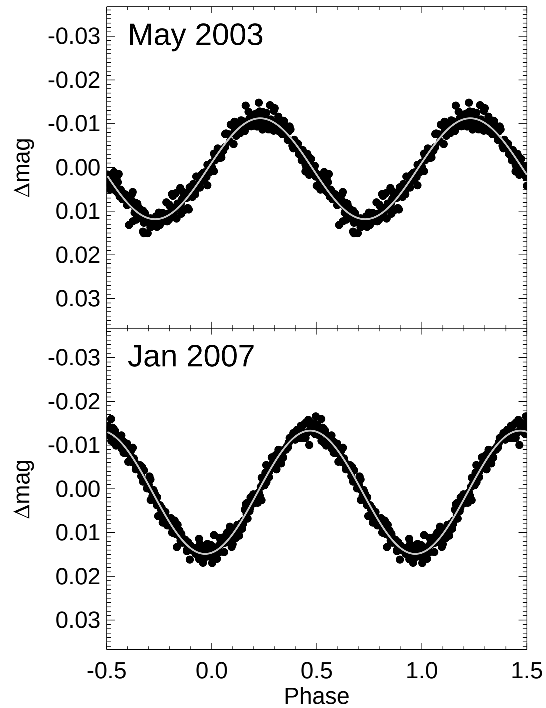

Figure 4. The phase folded light curves containing data from May 2003, and January 2007 highlight the excellent quality of data we have for computing the O-C residuals. One can clearly discern the increase in amplitude as well as the phase offset due to the changing period of Polaris, between the two light curves.

4.9 s yr^{-1}, they do follow a slightly shallower trend which is likely due to the recent decrease in period.

We will now discuss these results in the context of stellar evolution and ascertain what implications they have on the evolutionary stage of Polaris.

4 DISCUSSION

There is a great deal of evidence to suggest that Polaris is a first overtone (s-Cepheid) oscillator. Feast & Catchpole (1997) used the Hipparcos parallax (measured to be 7.56 ± 0.48 mas for Polaris) to fit Period-Luminosity models to a sample of Cepheids and concluded that the best fit for Polaris resulted if it was treated as a first overtone pulsator. The updated value for the Hipparcos parallax of Polaris is 7.54 ± 0.09 mas (van Leeuwen 2007; van Leeuwen et al. 2007), therefore this conclusion is still valid. Nordgren et al. (1999) used interferometry to measure the radius of Polaris to be $46 \pm 3R_{\odot}$, and again this is only consistent with the pulsation period if Polaris is a first overtone pulsator. The mean rate of change of the period and the small amplitude of the oscillations are also indicators that Polaris oscillates in an overtone mode. Combining this with the fact that Polaris exhibits a highly symmetrical light curve, it exhibits all of the features we expect from s-Cepheids.

Evans et al. (2002) suggest that Polaris exists at the cool edge of the region of the instability strip occupied by the s-Cepheids, but that the positive period change could not be due to evolution as the star would be evolving towards the centre of the instability strip, which would defy

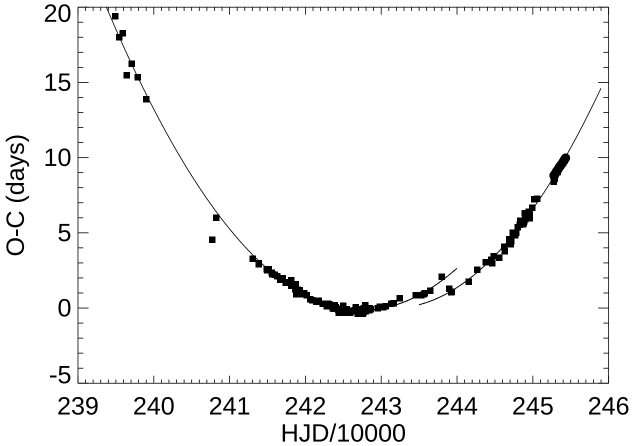

Figure 5. The O-C diagram for Polaris. We have combined SMEI results with archival data (see Turner et al. (2005) for a full list of references). The best fit parabola for the pre-1963 data is as in Turner et al. (2005), the parabolic fitting to the post 1965 data, including our new results at the right of the plot, gives a period change of 4.90 ± 0.26 s yr^{-1}.

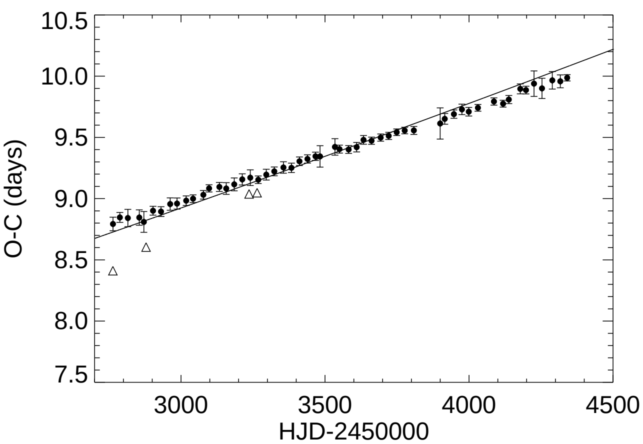

Figure 6. The O-C data obtained using SMEI, depicted by filled circles with error bars, along with the measurements from Turner et al. (2005), shown by triangles. Our measurements are fairly consistent with the 4.90 s yr^{-1} period change, but do appear to follow a slightly shallower trend.

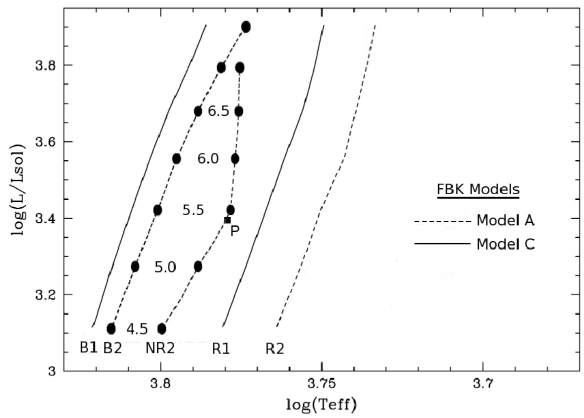

Figure 7. Theoretical models for the first overtone Cepheid instability strip (IS), computed by FBK. Results for convective model A (dashed lines) and convective model C (solid lines) are plotted. The labels B1 and B2 refer to the linear model blue edges for models A and C respectively, whilst R1 and R2 refer to the red edges. NR2 refers to the non-linear model red edge, and the values represent the masses used in the models. Plotting Polaris on the IS (filled square marked by P), we see it lies on the non-linear red edge boundary.

the previously declining amplitude. Turner et al. (2005) also place Polaris on the red edge of the instability strip for putative first crossers, which corresponds to the s-Cepheid red edge in this case. They do however suggest the possibility of Polaris being a fundamental pulsator, which is unlikely given the evidence above. Dinshaw et al. (1989) on the other hand believed that Polaris was about to evolve out of the instability strip completely, which would require it to be near the edge of the instability strip, which it certainly does not appear to be. The behaviour we are now seeing from Polaris is consistent with what we expect from a Cepheid located in the instability strip where Evans et al. (2002) and Turner et al. (2005) suggest, but the conclusion that the period change is not due to evolution may be incorrect.

Firstly, consider the evidence that the period has once again undergone a rapid decline, which could be a phase of blueward evolution, as Turner et al. (2005) suggests could be the case for the 1963-66 period readjustment. Secondly, the amplitude appeared to cease its decline in the early 1990s, and is now seemingly increasing again. One might expect to see such behaviour if Polaris was undergoing an evolutionary change in its oscillation mode. If we look at the models for s-Cepheids produced by Feuchtinger et al. (2000, hereafter FBK), and specifically look at the location of Polaris in the computed instability regimes then, as Fig. 7 shows, Polaris lies on the red edge of the overtone instability strip for their non-linear convective model, which is able to generate light and radial velocity curves very similar to those found observationally. If the true red edge of the first overtone instability strip lies close to the one computed in this FBK model, then Polaris is in fact undergoing a change from being a first overtone pulsator (assuming it is evolving to the cooler side of the instability strip) to becoming either a fundamental pulsator or a double mode pulsator. The position of Polaris in Fig. 7 suggests that its mass is not quite great enough to enter the fundamental pulsator regime, which occurs for masses greater than around 5.5 $M_{\odot}$ (the knee in the model track), but recent measurements by Evans et al. (2007) for example, do not place enough of a constraint on the mass to make an absolute determination as to which regime Polaris will enter. If Polaris is about to cross into another pulsation regime, then we might expect to see occasional blips in the period as this readjustment phase takes place.

Finally, the crossing mode of Polaris is also uncertain. Turner et al. (2005) suggest that despite Polaris exhibiting a deficiency of carbon and an over-abundance of nitrogen (Boyarchuk & Lyubimkov 1981; Luck & Bond 1986), this cannot be interpreted in any fashion to determine the crossing mode, such as was done by Kovtyukh et al. (1996). Somewhat controversially, Andrievsky et al. (1994) suggests that Polaris has a small over-abundance of carbon, which would certainly place it in a first crossing scenario.

The rate of period change for Cepheids is an indicator of the crossing mode (Turner et al. 2006). If we assume Polaris to be oscillating in the first overtone mode, and take the rate of period change over the last 150 years to be +4.5 s yr^{-1}, then this is a factor of $\sim$3 too small for a first crossing Cepheid and about the same factor too large for a third crossing Cepheid. Turner et al. (2005) suggest that the observable characteristics of Polaris are most consistent with a first time crosser. Interestingly, however, if one only considers the trend in the period from the last 20 years, then we see a decline at a rate of $\sim$ 10 s yr^{-1}. This is actually consistent with a first overtone Cepheid undergoing its second crossing. The possibility that Polaris is in its second crossing was also discussed by Engle et al. (2004). We must be cautious, however, as the rate over the last fourteen years, using measurements with much better precision than those presented in Dinshaw et al. (1989), only suggest a decline of $\sim$ 3 s yr^{-1}. Clearly, it is difficult to draw any firm conclusions on the crossing mode at present.

5 CONCLUSIONS

The new results obtained with SMEI strongly suggest the amplitude of Polaris is once again increasing. The star does seem to be oscillating in the first overtone mode, but likely lies close to the red edge of the instability strip for s-Cepheids, and is therefore likely to soon evolve into either a fundamental or double-mode pulsation Cepheid, assuming the Cepheid is undergoing its first or third crossing. It is uncertain how quickly Polaris will evolve across the boundary between pulsation regimes and what behaviour the Cepheid will exhibit as it does so. One slight oddity in the results is the lack of evidence for the fundamental mode of oscillation being present, but this may appear in the near future. It is therefore crucial that high precision monitoring of this star is continued for the foreseeable future so that the changes can be watched closely.

6 ACKNOWLEDGEMENTS

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